A Basic Model of Job Search

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Models of job search are prevalent throughout the economic literature, especially to study unemployment. This technical paper presents a basic model of job search. The model is analyzed to demonstrate the reservation wage property, where a worker will accept any wage offer above the reservation wage and will reject all other offers. The expected duration of unemployment for an unemployed worker is also analyzed. The model is then solved numerically using the Rapid Recursive® Toolbox. The numerical results show the reservation wage for a set of parameters and how a worker's reservation wage increases with increases to unemployment benefits.

I. Introduction

Previously, recursive models have been solved numerically using custom code.[1] The Rapid Recursive® Toolbox alleviates the need for custom coding by providing flexible tools to compose, check and solve recursive models.[2] To demonstrate the use of the Rapid Recursive® Toolbox, this technical paper presents a basic version of a canonical recursive model—the job search model—and solves the model numerically using the Rapid Recursive® Toolbox.

In addition to numerically solving the basic job search model, this paper analyzes the properties of the model analytically.

The rest of this technical paper is organized as follows:

- Section II outlines applications of the basic job search model.
- Section III develops the assumptions of the basic job search model and analyzes the properties of the model analytically, including the reservation wage property and expected duration of unemployment.
- Section IV outlines the parameterization of the model before it is solved numerically using the Rapid Recursive® Toolbox.
- Section V summarizes the results, including how a worker's reservation wage responds to changes in unemployment benefits.
- Section VI lists a number of extensions to the basic job search model.
- Section VII provides concluding remarks.

1. Recursive models are also called Markov decision problems or dynamic programming problems.
2. The Rapid Recursive® Toolbox is produced by Supported Intelligence, LLC and is a toolbox that runs on top of MATLAB®. Supported Intelligence, LLC provides advanced analytics for investment modeling, valuation, financial modeling, risk assessment, and decision optimization. For more information, see: www.supportedintelligence.com.

II. Job Search Models

Versions of the basic job search model are prevalent throughout the economic literature, especially to study unemployment. A survey of the vast theoretical literature on job search is presented by Rogerson, Shimer and Wright (2005). For an empirical perspective of the the job search literature, see Divine and Kiefer (1995).

In addition to studying unemployment, many other interpretations of the basic job search model are possible and have been used throughout the literature, including the problems of a person looking for a partner ("optimal stopping problem"), a buyer looking for a house, and an investor searching for investment opportunities.

Supported Intelligence, LLC has applied the job search model in two different contexts: the problem of a rental property manager searching for a tenant, and the owner of an asset deciding when to sell it. The rental property manager application can be found in the Rapid Recursive® Toolbox, and the sale of an asset application is discussed in the User's Guide to the Rapid Recursive® Toolbox.

III. Model

In the model[3], an unemployed worker receives a wage offer for employment, $w_0 \in [w_{lower}, w_{upper}]$. Wage offers are drawn independently from a discrete truncated normal distribution, $F(\bar{w}) = \text{Prob}(w \le \bar{w})$. The worker can choose to accept or reject the offer. If the worker accepts the offer they receive that wage every period forever (the worker is infinite-lived). If the worker rejects the offer, they receive unemployment benefits c for that period; in the following period they receive another (independent) wage offer for employment and the process repeats as before.

The unemployed worker must choose whether or not to accept the wage offer they receive. The unemployed worker's goal is to maximize their expected lifetime discounted stream of income, with

3. The model developed borrows from lecture notes by Professor Matthias Doepke for ECON411-3 (a Ph.D. level macroeconomics course at Northwestern University) as well as Ljungqvist and Sargent (2004).

discount factor, $\beta \in (0, 1)$. That is, the worker's objective function is:

$$E_0\left[\sum_{t=0}^{\infty} \beta^t y_t\right]$$

where $y_t = w_t$ if the worker chooses to accept the wage at time t; and $y_t = 0$ otherwise. Since the model is stationary, we can drop the time subscripts henceforth.

The wage w can be interpreted as representing the overall attractiveness of a job, including the wage, location of the job, prestige of the position, social benefits of the job etc.

Similarly, the unemployment benefits c can be interpreted more broadly. The variable c can be interpreted to be the value of not working, including the value of leisure associated with not working, the cost of job search and any other income associated with not working.

It turns out that the infinite time assumption in this model is not too limiting because it can be thought of as an approximation of a finite time model with a long time horizon.

RESERVATION WAGE PROPERTY

Before solving the model numerically, a few features of the model are explored analytically.

If the worker accepts their offer they receive:

$$w + \beta w + \beta^2 w + \beta^3 w + \ldots = \frac{w}{1 - \beta}$$

Let's call this value $V(w)$ and refer to it as the value of employment:

$$V(w) = \frac{w}{1 - \beta} \qquad (1)$$

Let U be the value of not accepting the offer. When a worker does not accept the offer, they receive c plus the value of receiving another offer in the following period. The value of receiving another offer in the following period is equal to $V(w)$ if they accept the offer, and is equal to U if they do not accept the offer. Therefore,

$$U = c + E[max(V(w), U)] \qquad (2)$$

With these preliminaries, the nature of the solution to this model can be characterized: there is a value R for which a wage offer less than R will be rejected by the worker, an offer above R will be accepted, and the worker is indifferent between accepting and rejecting an offer of R. R is referred to as the reservation wage.

The "proof" of this reservation was is as follows. The value of employment $V(w)$ is an increasing function of w, as can be seen from equation 1. On the other hand, it can be seen from equation 2, that U is independent of w (the expectation term is just a constant). Using these facts, $V(w)$ and U are plotted on a graph in Figure 1. R is the wage at which the two lines meet. For each wage offer, the worker chooses the higher $V(w)$ and U. From the graph, you can see that when $w < R$, the graph of U lies *above* $V(w)$ so the worker will reject the wage offer. Likewise, when $w < R$, the graph of U lies *below* $V(w)$ so the worker will accept the wage offer.

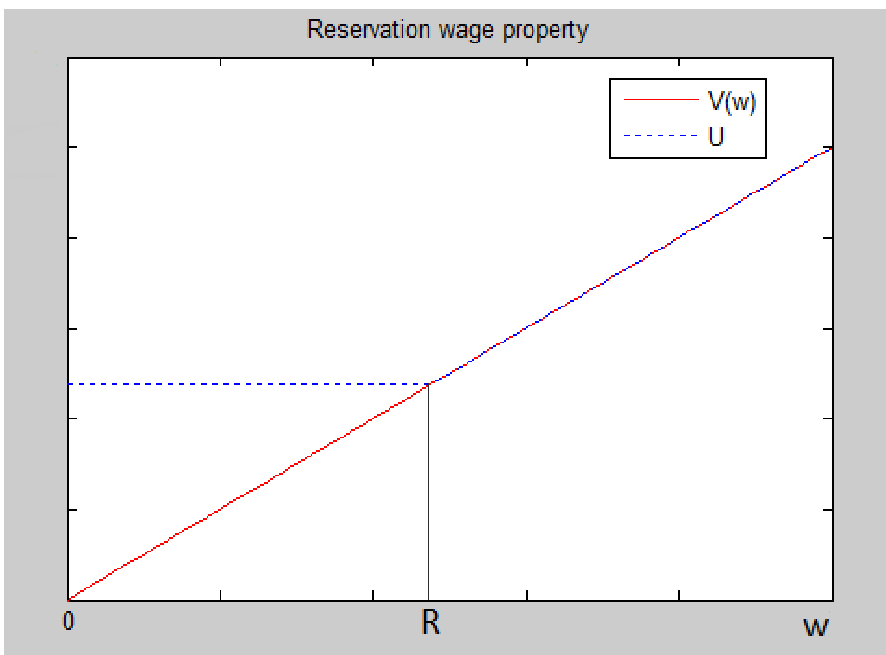

Figure 1: Reservation Wage Property

EXPECTED DURATION OF UNEMPLOYMENT

The "hazard rate", which is the probability of an unemployed worker accepting a job, is: $H = 1 - F(R)$. Letting D be the duration of unemployment, a bit of algrebra can be used to show that the expected duration of employment is:

$$E[D] = \Sigma_{d=1}^{\infty} d(1-H)^{d-1} H = \frac{1}{H} \quad (3)$$

(The $(1-H)^{d-1}H$ term inside the summation is the probability that the worker remains unemployed for exactly d periods, i.e. it is the probability that the worker will receive $d-1$ offers below their reservation wage followed by an offer above their reservation wage).

From equation 3, it can be seen that increasing c raises the expected duration of unemployment (because raising c will raise U which raises R and lowers H). Also, it is interesting to note that raising c increases the utility of the worker since they have more utility when they are unemployed, even though they stay unemployed longer.

IV. Solving The Model

The model is solved numerically using the Rapid Recursive® Toolbox. The Rapid Recursive® Toolbox contains tools for composing, checking and solving recursive models such as the basic job search model. The Rapid Recursive® Toolbox is produced by Supported Intelligence, LLC and is a toolbox and runs on top of MATLAB®. When solving the model, the following parameterization was used:

TABLE 1: Parameterization

Parameter	Value
w_{lower}	\$17,000
w_{upper}	\$40,000
β	0.9
c	\$10,000 to \$20,000
$F(w)$	Discrete, truncated normal with mean \$28,500 and standard deviation 3,833.
Frequency of periods	Yearly

Further, the wage variable is estimated by a grid with 1001 points. The model is run for values of c between \$10,000 and \$20,000 to see how the reservation wage responds.

The parameterization of the model is more or less arbitrary and for demonstration purposes only. However, a bit of interpretation is possible. The wages can be interpreted as yearly salaries. The value of w_{lower} was chosen to resemble the salary of a worker on the minimum wage. And the value of c was chosen to resemble the benefits of an unemployed person.

The code for this model was written by Ariell Zimran, a Ph.D. candidate in economics at Northwestern University. The model is solved using the policy iteration algorithm included in the Rapid Recursive® Toolbox.

It should be noted that the sample code is just one way to set up this model numerically. The "principle of optimality" can be used to halve the number of states that need to be set up.

The sample code defines the states as all combinations of whether a worker is employed or not and the wage offer they receive, which is technically correct. However, using the "principle of optimality", we know that a wage offer of w_t that was accepted at time t will also be accepted at time $t + 1$, $t + 1$ etc. Therefore, the states in the code can just be defined as the wage offer that the worker receives each period. The transition matrix then needs to make sure that if the worker accepts an offer, they will continue to receive the same wage offer (and will continue accepting this offer), and if the worker rejects the offer, then the worker will receive a completely new offer in the following period.

V. Numerical Results

Figure 2 shows how the reservation wage increases with the value of c. A worker increases the minimum wage they are willing to accept if they have higher benefits from not working.

Figure 2: Reservation wage versus unemployment benefits

VI. Extensions

Many extensions of the basic job search model are possible. Just a few of them are listed:

- Workers and employers can have exogenous separations, which can be interpreted as workers quitting or getting laid off.
- The number of offers the unemployed worker receives each period can increase with how intensively they search.
- Workers can job search while employed and then choose to switch between jobs (Burdett, 1978) (this endogenizes quits).
- There can be cyclical fluctuations in job offers as well as on-the-job search. In these models, workers are more likely to quit when the economy is strong because they can receive better wage offers.
- Workers can learn about the characteristics of the job as they work, e.g. the worker may not know how much they enjoy the job until they start working. The uncertainty here causes workers to accept jobs they wouldn't otherwise accept and vice versa.
- The model can be set over a finite time horizon.
- The model can be set in continuous time. This is a common assumption in "matching" models.

VII. Conclusion

A basic job search model was presented and solved numerically using the Rapid Recursive® Toolbox. The Rapid Recursive® Toolbox provides a simple avenue for analyzing and solving the basic job search model. Although the model presented is simple, a number of variations of the model are common throughout the labor economics literature.

VIII.References

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