Valuing Rental Properties Using a Recursive Model

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Rental properties are traditionally valued using the sale price of comparable properties or by calculating the discounted cash flow of future rental income. These methods, however, have known shortcomings when valuing rental properties. This technical paper presents a "recursive method" approach to valuing rental properties. The recursive method is an improvement to the discounted cash flow method because it allows conditions to change and for the property owner to adapt to those changes. The recursive method is then used to calculate the value of an actual rental property in the state of Michigan in the United States. Data used in the valuation model is provided by the property's owner.

I. Introduction

Rental properties are traditionally valued using the sale price of comparable properties (the "market approach") or by calculating the discounted cash flow of future rental income (the "income approach"). These methods, however, have shortcomings when valuing rental properties.

The market approach may produce an estimate that is divorced from the underlying value of the rental property. This drawback can occur, for example, when comparable properties are purchased for particularly high prices due to the idiosyncratic preferences of an individual. Valuations that are not based on the underlying value of an asset are associated with asset bubbles. A further shortcoming of the market approach is the potential lack of comparable properties, especially for unique properties or properties in market areas with low trading volume.

While the discounted cash flow method alleviates these problems with the market approach, it introduces other problems. For example, the discounted cash flow does not recognize that rental income can unexpectedly change in the future.

This technical paper presents a novel approach to valuing rental properties. This approach uses a recursive method, which is also known as dynamic programming. The recursive method is similar to the discounted cash flow method in that it discounts future cash flows, but it differs in that it recognizes that rental income can unexpectedly change in the future and allows rental property owners (or rental property managers) to adjust to these changes.

The recursive method is then used to estimate the value of an actual rental property in the state of Michigan in the United States. Data used in the valuation model is provided by the property's owner.

The model developed in this paper also helps to answer the following questions: Should I buy the property? Or if you own the property: should I sell the property? At what price level should a property owner rent out the property for? Is it better to wait a month to rent the property if only low offers are received?

The rest of this paper is organized as follows:

- Section II develops a general model of a rental property valuation using a recursive method.

- Section III applie's the general model to value a property in the state of Michigan in the United States.
- Section IV outline's how the models are created and solved using the Rapid Recursive® Toolbox.
- Section V presents the results.
- Section VI provides concluding remarks.

II. The Recursive Model

The general property valuation model developed in this paper is an extension of the basic job search model, where an unemployed worker is searching for a job (see the Supported Intelligence technical paper (2013). In this sense, the model developed in this paper can be thought of as a rental property owner searching for a tenant.

The major differences between the model developed here and the basic job search model is that rental offers follow a different stochastic process and exogenous separations are possible in the rental property model.

The general model developed in this paper is represented diagrammatically in Figure 1 and Figure 2. Figure 1 is a decision tree representation of the model, while Figure 2 is a recursive representation. Both diagrams are equivalent.

Figure 1: Decision Tree Representation of Model

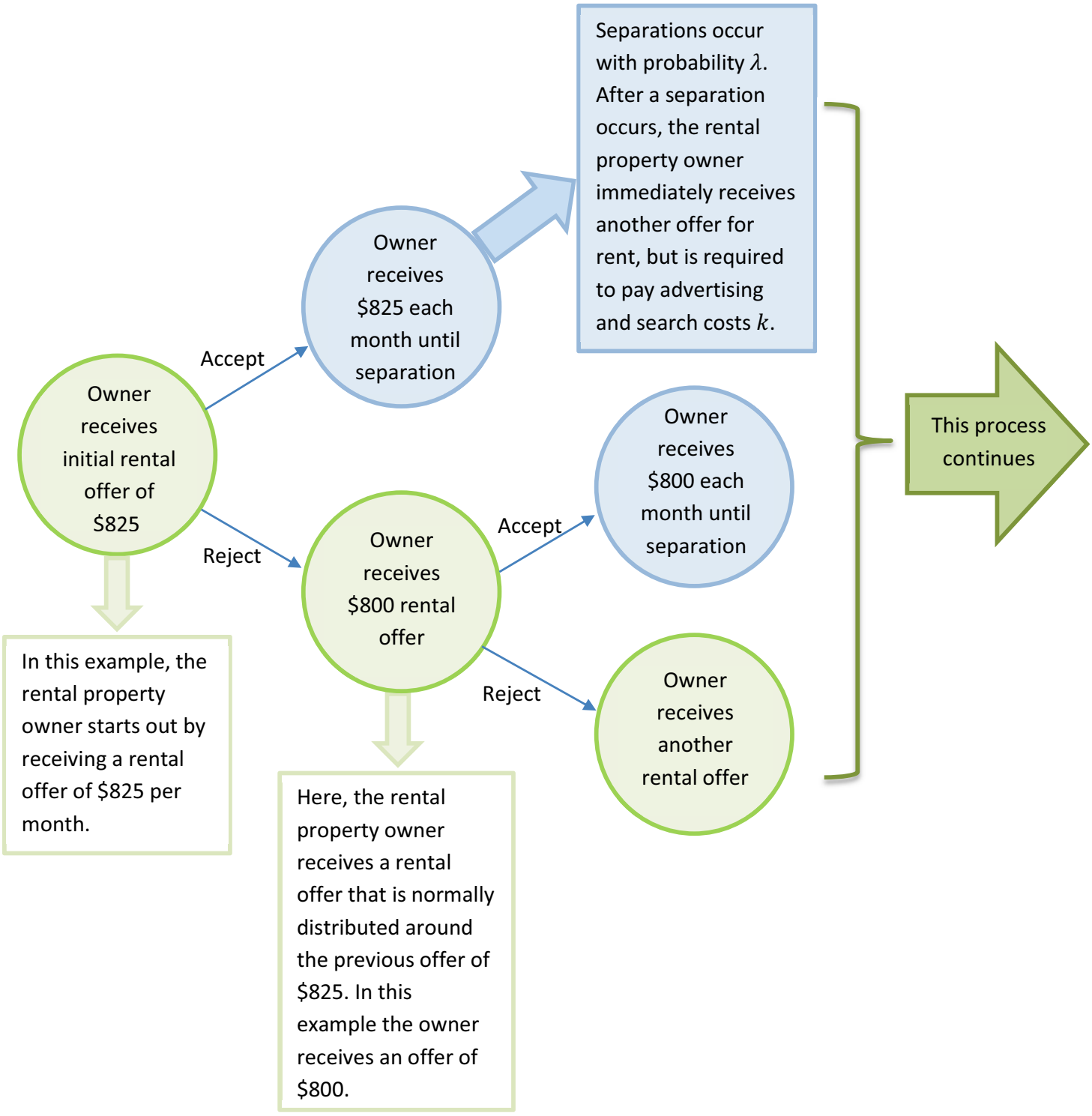
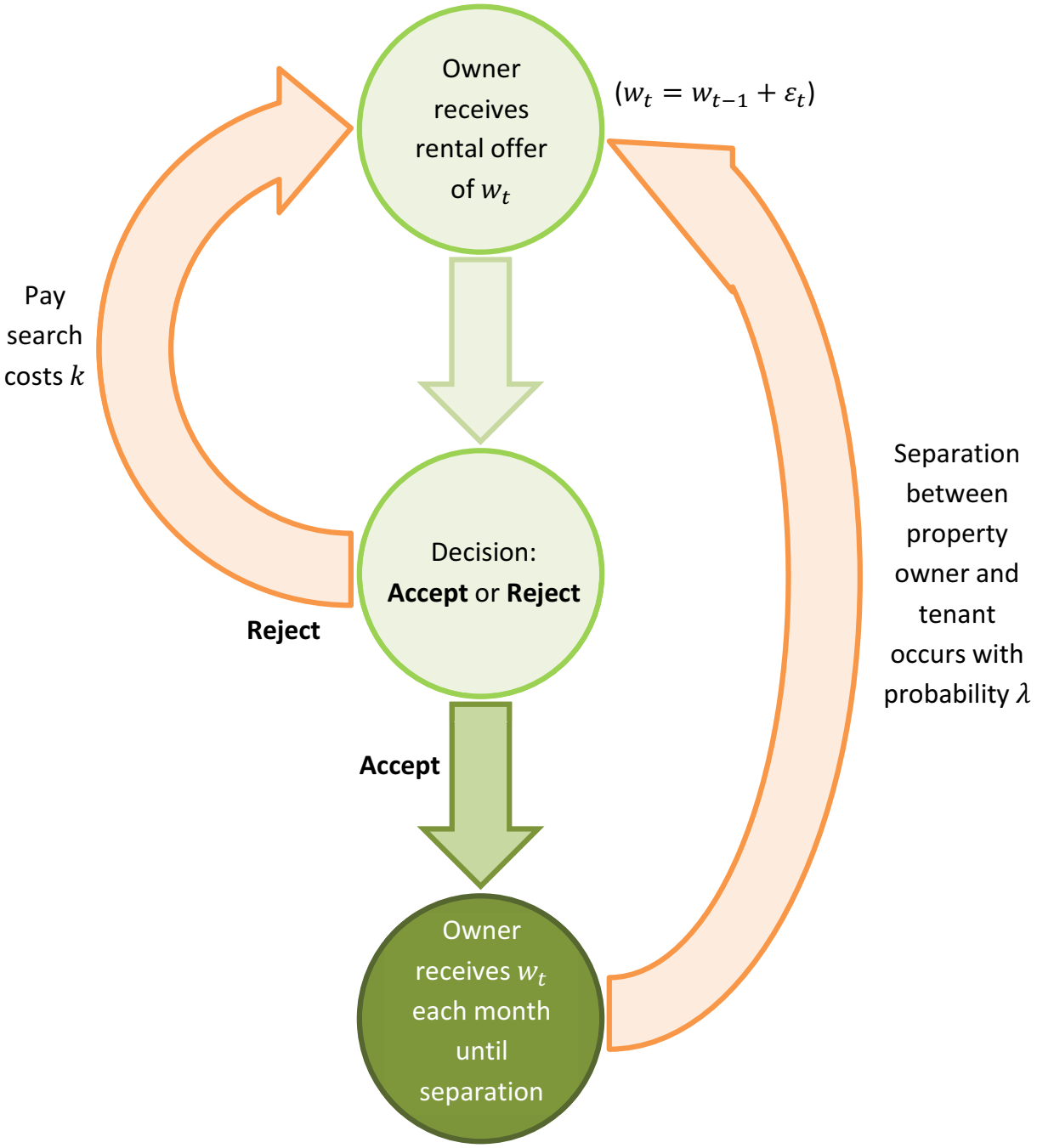


Figure 2: Recursive Representation of Model



In the model, the rental property owner (this may be a rental property manager) has received an offer $w_0 \in [w_{lower}, w_{upper}]$ to rent their property. The rental property owner can choose to accept or reject this offer.

On the other hand, if the rental property owner rejects the offer, they do not receive any rental income for that period and must pay advertising and search costs k to find another potential tenant; in the period following the rejection, the property owner receives another rental offer w_t that is drawn from a Brownian motion process as follows: $w_t = w_{t-1} + \varepsilon_t$, and $w_0 \in [w_{lower}, w_{upper}]$ for $t \geq 0$, and ε_t is (discrete, truncated) normally distributed with zero mean and variance σ^2. This process repeats: the rental property owner can choose to accept or reject this offer of w_t, and so on.

A separation does not lead to a period where the rental property owner does not receive any rental offers. Instead, in the period where a separation occurs, the rental property owner continues to receive rental offers according to the Brownian motion process listed above, with one exception: in the period where a separation occurs (and only in this period) the Brownian motion process is centered around the last rent the rental property owner was receiving before the separation, i.e. $w_t = lastrent + \varepsilon_t$.

In the period where a separation occurs, since the rental property owner is searching for another tenant, they are required to pay advertising and search costs k regardless of whether the property is immediately rented out again or not.

The rental property owner also has costs F that do not depend on whether the property is occupied by a tenant. These include: property taxes, insurance, legal and accounting costs, and homeowners assocation fees.

In addition, the rental property owner has costs $c(x)$ that vary depending on whether the property is occupied $x = 1$ or not $x = 0$. These costs include maintenance costs.

The owner of the rental property owner must choose whether or not to accept each rental offer they receive with the goal of maximizing their expected lifetime discounted stream of income, with discount factor, $\beta \in (0, 1)$. That is, the rental property owner's objective function is:

$$E_0\left[\sum_{t=0}^{\infty} \beta^t \gamma_t\right]$$

Equation 1:

where:

$$y_t = \begin{cases} w_t - c1(1) - F & \text{if the property if occupied (except during a separation)} \\ -k - c(0) - F & \text{if the property is vacant} \\ w_t - c(1) - F & \text{if a separation has just occured but a new tenant if found} \end{cases}$$

BRIEF DISCUSSION OF RECURSIVE METHOD

The similarity between the discounted cash flow method and the recursive method can be seen by expanding out equation 1:

$$E_0\left[\sum_{t=0}^{\infty} \beta^t y_t\right] = E_0[y_0 + \beta y_1 + \beta^2 y_2 + \beta^3 y_3 + \ldots]$$

From the right hand side of the equation, you can see that the recursive method is discounting a stream of cash flows—discounting is being applied to the cash flows y_1, y_2, y_3 etc. In this sense, the recursive method is similar to teh discounted cash flow method.

The difference between the methods is that in the recursive method, the actual cash flow y_t that is discounted can change if conditions change and if the property owner adjusts to the changing conditions.

For example, in this model the owner of the rental property can choose to reject a rental offer if it is too low and instead wait for another offer in the following period. In this example, a rejected rental offer at time t leads to a low value of y_t, while an accepted offer at time t leads to a higher value of y_t.

By allowing the rental property owner to adapt to changing conditions, this model values the "real option" of the rental property owner to wait for better rental offers when s/he only receives a low rental offer.

The advantages and disadvantages of using recursive models are described in a number of references including Dixit and Pindyck (1994) and Anderson (2013). For an in-depth discussion of recursive models, the reader should refer to these books.

III. Valuing a Property in Michigan

The general property valuation model developed in the previous section is applied to value a property located in the state of Michigan.

DESCRIPTION OF THE PROPERTY

The property is a 1,000 square foot townhouse style unit in the East Lansing / Lansing area in Michigan. It has 2 bedrooms and 1.5 bathrooms. It is a short drive from the city of Lansing, the capital of Michigan.

SURVEY

The owner of this property was interviewed and asked to fill out a questionnaire about their property. Based on the answers to the questionnaire, the general model was parameterized as shown in Table 1. Further, the rent variable is estimated by a grid with grid points spaced at $5 increments.

TABLE 1: Parameterization of Model

Parameter	Value (per month)	Source
Initial rental offer w_0	$825.00	Data from questionnaire
w_{lower}	$660.00	Modeling assumption
w_{upper}	$990.00	Modeling assumption
$c(0)$	$145.00	Data from questionnaire
$c(1)$	$193.33	Data from questionnaire
k	$560.00	Data from questionnaire
F	$547.50	Data from questionnaire
λ	0.0417	Based on an average length of tenancy of two years
β	0.9988	Based on 3.5% mortgage and 2% inflation
σ^2	2500	Modeling assumption
Frequency of periods	Monthly	Modeling assumption

Source: Information provided by the property owner and judgment of the author.

DISCUSSION OF PARAMETERS

The value of the initial rental offer (w_0) in the model is sourced directly from the questionnaire completed by the property owner. The upper and lower bounds are modelling assumptions made by the author.

The costs $c(0)$ and $c(1)$ are based on the actual costs paid by the property owner: property taxes, insurance, legal and accounting costs, and homeowners assocation fees.

The search and advertising costs k are estimated based on the property owner's (or more likely the property manager's) time showing the apartment and the use of Craigslist. The use of Craigslist is free and is typical in this market area. (For more densely populated market areas, it is more common to use an apartment search agency). The property owner's time is estimated at $35 per hour, and the property owner estimated that searching for a tenant takes about 16 hours of their time per month of searching.

The fixed costs of the property include: insurance, accounting and legal fees, home owners association fees, depreciation and property taxes. With the except of depreciation, all of these costs are the actual costs of the property owner. Depreciation is estimated by the author of this paper by separating the building's cost from the value of the land. This calculation is made based on Michigan's average land-to-value ratio data from the Lincoln Institute of Land Policy.

The chance of a separation λ is based on an average tenancy length of 24 months, which is based on both data from the questionnaire. That is, $\lambda = 1/24$.

The discount factor β is calculated based on the property owner's actual annual mortgage rate of 3.5% and an estimated annual inflation rate of 2%. These annual rates are converted into monthly rates. The discount factor is then calculated using the following formula: $\beta = \frac{1+g}{1+d}$, where d is the monthly mortgage rate and g is the monthly inflation rate. The reason the mortgage rate is used is because that is the cheapest cost of capital available to property owners.

The property owner's mortgage rate is consistent with current market mortgage rates of between 2.5 to 3.5% for 15- and 30-year fixed-rate mortgages.

IV. Solving the Model

The model is solved numerically using the Rapid Recursive® Toolbox. The Rapid Recursive® Toolbox contains tools for composing, checking and solving recursive models such the one presented in this paper.

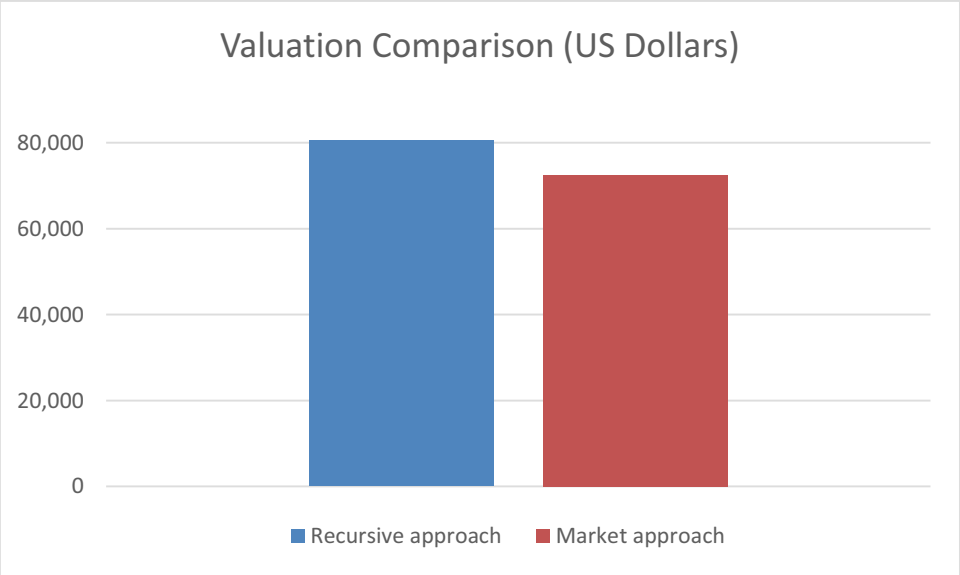
The model was created by adjusting the "Rental Property Management" Solution Template that is included in the Rapid Recursive® Toolbox. The model was solved using the policy iteration algorithm also included in the Rapid Recursive® Toolbox.

To run the Rapid Recursive® Toolbox, MATLAB® (by The MathWorks) is required. In addition, functions from the MATLAB® Statistics Toolbox were used.

V. Results

The value of the property is estimated by the recursive model to be $80,517.58. This compares with an estimated market value of the property of between $70,000 to $75,000 based on the property owner's assessment of recent market transactions of comparable properties. These results are summarized in Figure 3 and Table 2.

Figure 3: Comparisn of Valuation Estimates



The fact that the recursive model estimates a value that is higher than the market value suggests that property owner is better off keeping the property as a rental property than selling it.

The results of the model also estimate the rental property owner's reservation rent as $785. This means that is is optimal to reject any offer below $785, and wait for another offer in the following period. If the property owner receies a rental offer equal to of above $785, it is optimal to accept that offer.

TABLE 2: Results

Recursive Valuation	$80,517.58
Market Value*	$70,000 to $75,000
Reservation Rent	$785
Average Vacancy	0.0211

*Market value is based on the property owner assessment of recent market transactions of comparable properties.

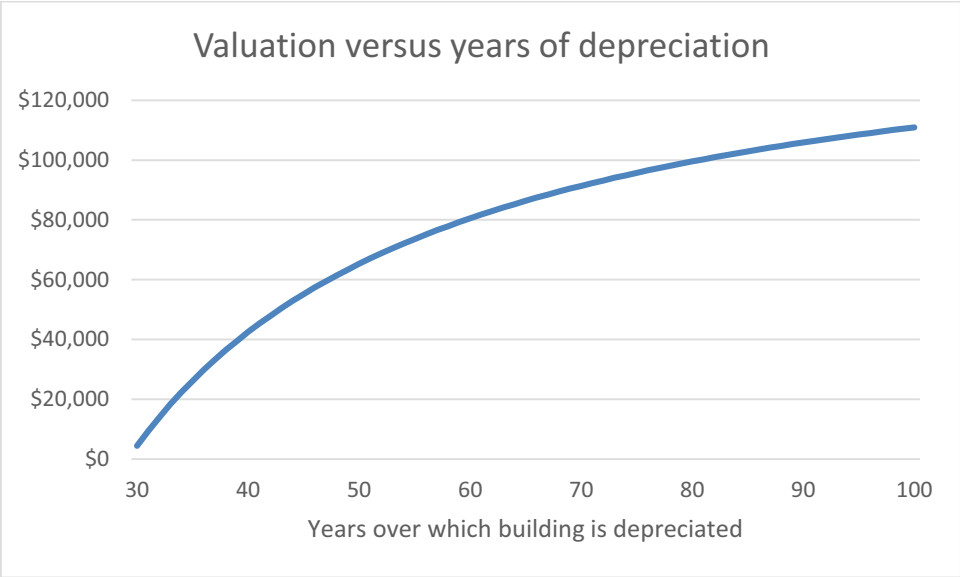
The average vacancy is the proportion of time the property remains vacant due to a low rental offer. The average vacancy in the model of 0.0211 equates to the property being vacant for one month approximately every four years. This result is consistent with data provided by the property owner.

SENSITIVITY TESTING

For illustrative purposes, sensitivity testing is conducted to see how the value of the rental property changes when the number of years over which the building value is depreciated over changes. The results of the sensitivity testing show how the value of the rental property increases as the building is depreciated over a larger number of years, as can be seen in Figure 4.

The results above were generated assuming that the value of the building is depreciated over 60 years. When the building is depreciated over 54 years instead, the estimated value of the rental property value falls below the market value.

Figure 4: Sensitivity Testing



VI. Conclusions

In this paper, a general model for valuing rental properties using a recursive method is presented. The recursive method is then used to calculate the value of an actual property located in the state of Michigan in the United States.

The value of the rental property is estimated to be higher than the market value of the property. Although these results suggest that the property owner is better off keeping the property as a rental property than selling it, the results are sensitive to a few assumptions in the model. For example, when the value of the building is depreciated over 54 instead of 60 years, the recursive model estimate of the value of the rental property falls below the market value. As such, while the results present a case that the value of the rental property is higher than its current market value, such a conclusion hinges on the investor's opinion about the value of assumptions in the model, such as depreciation.

Further, the property owner should make their "buy or sell" decision in the context of the risk and reward characteristics of their whole investment portfolio, as well as the other investment opportunities that they may have available.

VII. References

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